Optimal management of public real estate under seismic risk using real option approach

M. Dan\textsuperscript{1} and M. Kohiyama\textsuperscript{2}

**ABSTRACT**

This study focuses on the scheduling in public real estate management (PREM) and derives the optimal scheduling using real option approach considering seismic risk. This study defines a seismic risk as the uncertainty in PREM and proposes a method to derive the optimal scheduling of seismic retrofitting of facilities managed by a municipality using real option considering seismic risk. To verify the effectiveness of the proposed method, this study conducts a case study. A retrofitting problem of school facilities in Kawasaki City, Japan is selected as a case study subject. Through the case study, it is confirmed that the proposed method can derive the optimal retrofit schedule with a reduced cost and a reduced risk of damage due to earthquakes.

**Introduction**

Recently, the importance of the public real estate management (hereafter, PREM) is emphasized not only in academia but also in national and local governments because they need to manage their facilities appropriately with their limited budget. This study focuses on the scheduling in PREM and derives the optimal schedule using real option approach (hereafter, ROA) considering seismic risk. The public real estates are usually used for a long time and involve large uncertainty in its management. The risk of earthquakes and other natural disasters should be considered in PREM. This study defines a seismic risk as the uncertainty in PREM and proposes a method to derive the optimal retrofit schedule of public facilities.

**Modeling of seismic risk using risk curves**

In this study, a seismic risk is represented by risk curves using an $I_s$ value, which is a seismic performance index used in Japan. A secular variation model of building performance represented by the $I_s$ value, $I_s(t)$, is constructed by a regression analysis using data of 115 buildings of public school in Kawasaki City, Japan. The risk curve is evaluated with a loss curve derived from fragility and hazard curves; a fragility curve represents the seismic vulnerability of a building, and a hazard curve represents a seismic risk at a site location. The risk curve is defined as a probability $P_R$ that the loss of objective buildings due to earthquakes exceeds $C_D$ at least once in a time period $\Delta t$ from the time $t$ as Eq. 1:

$$P_R(C_D, t, \Delta t) = P_H(Y > y; \Delta t)$$  \hspace{1cm} (1)

\textsuperscript{1}Research Assistant, Keio University, Kanagawa, Japan
\textsuperscript{2}Associate Professor, Keio University, Kanagawa, Japan
where $y$ represents the earthquake intensity that causes the damage $C_D$. The proposed method generates $C_D$ with the inverse function of the risk curve with a random number of $p$, which follows uniform distribution in $[0, 1]$, as shown in Eq. 2 for a Monte Carlo simulation.

$$C_D(t, \Delta t) = P_R^{-1}(p) \quad (2)$$

**Real option approach considering seismic risk**

ROA is a project evaluation method, which takes a place of the NPV method [1]. The greatest feature of ROA is an ability to consider uncertainties in the future. This study regards a seismic risk as future uncertainties and evaluates real option by using a binomial option pricing model [2] with switching option [3]. We define a stochastic variable $Z_t$ as the changing rate of an underlying asset value $S(t)$ at the time $t$. Here, $Z_t$ can be defined as Eq. 3:

$$Z_t = \begin{cases} 
  u(t) = d(t) + \frac{(1 + r) - d(t)}{1 - P_R(t, \Delta t)}, & 1 - P_R(C_D, t, \Delta t) \\
  d(t) = 1 - \frac{C_D(t, \Delta t)}{S(t)}, & P_R(C_D, t, \Delta t)
\end{cases} \quad (3)$$

where $u(t)$, $d(t)$, and $r$ are the increasing rate of $S(t)$, the decreasing rate of $S(t)$, and the discount rate, respectively. $u(t)$ is defined in the same manner as the risk-neutral probability.

When there are $m$ stages for the switching option, which $S(t)$ can select at the time $t$ ($t = 0, 1, ..., T$), the switching cost from the stage $s_B$ ($s_B = 1, 2, ..., m$) (the subscript $B$ means “Before”) to the stage $s_A$ ($s_A = 1, 2, ..., m$) (the subscript $A$ means “After”) is defined as $C_{s_A, s_B}(t, S(t))$. The project value after the decision making at the expiry ($t = T$), $V_A$, can be calculated with the cash flow $CF$ as Eq. 4:

$$V_A(T, S(T), s_A) = CF(T, S(T), s_A) \quad (4)$$

The project value before the decision making at the expiry ($t = T$), $V_B$, can be calculated using $V_A$ as Eq. 5:

$$V_B(T, S(T), s_B) = \max_{s_A} \{V_A(T, S(T), s_A) - C_{s_B, s_A}(T, S(T))\} \quad (5)$$

$V_A$ at the time $t$ ($t = 0, 1, ..., T - 1$) can be calculated as the sum of following two values: i) cash flow at the time $t$ and ii) the discounted project value at the time $t + 1$ as Eq. 6:

$$V_A(t, S(t), s_A) = CF(t, S(t), s_A) + \frac{1}{1 + r} E_t^0[V_B(t + 1, S(t + 1), s_B)] \quad (6)$$

$V_B$ can be calculated similarly in Eq. 7:

$$V_B(t, S(t), s_B) = \max_{s_A} \{V_A(t, S(t), s_A) - C_{s_B, s_A}(t, S(t))\} \quad (7)$$
where $E_t^Q[\cdot]$ denotes the expected value and $s_A|t = s_B|t+1$ holds constantly. Finally, $V_B(0,S(0),s_B)$, the project value before the decision making on the stage $s_B$ at the time $t=0$ can be calculated by using Eq. 4-7 repeatedly.

**Proposal of a schedule optimization method for PREM**

The ultimate purpose of PREM is “the total optimization of the management for all the public facilities in a specific area considering various options, such as relocation, reorganization, and composition of facility functions”. This study aims to find the optimal selection at each stage to maximize the total project value of the facilities before the decision making at $t=0$. The objective function is defined as Eq. 8 with the total value $S_{all}$ including all facilities:

$$
\text{Maximize } J = \frac{1}{N_{mc}} \sum_{n=1}^{N_{mc}} V_B(0,S_{all}(0),s_B(0)) - \sum_{i=1}^{N_R} \left( \sum_{t=0}^{T} x_{t,i} \right) 
$$

where $N_{mc}$ and $x_{t,i}$ are the number of samples in the Monte Carlo simulation and the decision variable in the optimization. $N_R$ is the number of facilities. When the budget for the facility $i$ at the time $t$ is defined as $C_{b,i}(t)$, the decision variable matrix can be represented as Eq. 9:

$$
\mathbf{x}(t) = \left[ C_{b,1}(t), C_{b,2}(t), ..., C_{b,N_R}(t) \right] 
$$

Constraints are introduced to consider the limitation of the budget. The total of the switching costs $C_{all}$ is lower than the sum of the budget $C_b$ and the cash flow $CF$. The constraint inequalities are formulated as Eq. 10:

$$
\begin{align*}
\text{Subject to } & \left\{ C_{all}(t,S_{all}(t)) \leq C_b(t) + CF(t,S_{all}(t),s_A(t)) \right. \\
& 0 < C_{b,i}(t) < C_b(t) 
\end{align*}
$$

**A case study and verification of effectiveness of the proposed method**

A case study is conducted for a retrofitting project for a group of 3 school facilities ($N_R = 3$) in Kawasaki City, Japan. The risk curve at $t = 0$ is shown in Fig. 1.
Fig. 2 shows the optimization result, which represents the optimal schedule for 20 years using 1000 samples in Monte Carlo simulation. Stage 1 indicates no retrofitting, and stage 2 and 3 indicate the condition with the $I_s$ value of 0.7 and 0.9 after retrofitting works, respectively.

![Figure 2. The optimization result.](image)

The comparison of project values and seismic damages between the proposed method and the NPV method are shown in Table 1. It is confirmed that the proposed method can not only increase the project value but also reduce the seismic risks.

<table>
<thead>
<tr>
<th></th>
<th>Project value $V_B(0, S(0, s_B))$</th>
<th>Expected seismic damage</th>
<th>Maximum seismic damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed</td>
<td>369.2</td>
<td>0.1144</td>
<td>19.34</td>
</tr>
<tr>
<td>NPV</td>
<td>366.3</td>
<td>0.1995</td>
<td>21.45</td>
</tr>
</tbody>
</table>

**Table 1. Project values and Seismic damages.**

**Conclusions**

In this study, it is confirmed that the proposed method can derive the optimal retrofit schedule of public facilities with a reduced cost and reduced seismic damage risks. The risk curve employing an $I_s$ value is used to represent the seismic risks of facilities. The seismic risk is newly considered as one of uncertainties in PREM by using the risk curve.

**Acknowledgments**

This work is supported in part by a Grant-in-Aid for the Leading Graduate School program for “Science for Development of Super Mature Society” from the Ministry of Education, Culture, Sport, Science, and Technology in Japan.

**References**